MENIIT

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JEE MAIN-2021

COMPUTER BASED TEST (CBT)

DATE : 27-08-2021 (EVENING SHIFT) | TIME : (3.00 pm to 6.00 pm)

Duration 3 Hours | Max. Marks : 300

QUESTION & SOLUTIONS

PART A : PHYSICS

Single Choice Type

This section contains **20 Single choice questions**. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

1. For full scale deflection of total 50 divisions, 50 mV voltage is required in galvanometer. The resistance of galvanometer if its current sensitivity is 2 div/mA will be:

(1) 1 Ω (2) 2 Ω (3) 5 Ω (4) 4 Ω

Ans. (2)

Sol. $C.S. = \frac{\theta}{I_g}$

$I_g = \frac{\theta}{C.S.} = \frac{50div}{2div/mA} = 25mA$

Potential difference = 50 mV

Resistance $= \frac{V}{I} = \frac{50mV}{25mA} = 2\Omega$

2. For a transistor α and β are given as $\alpha = \frac{I_C}{I_E}$ and $\beta = \frac{I_C}{I_B}$. Then the correct relation between α and β will be :

(1) $\alpha = \frac{1-\beta}{\beta}$ (2) $\alpha\beta = 1$ (3) $\beta = \frac{\alpha}{1-\alpha}$ (4) $\alpha = \frac{\beta}{1-\beta}$

Ans. (3)

Sol. $i_e = i_b + i_c$

$\frac{i_e}{i_c} = \frac{i_b}{i_c} + 1$

$\frac{1}{\alpha} = \frac{1}{\beta} + 1 \quad \Rightarrow \beta = \frac{\alpha}{1-\alpha}$

3. If the rms speed of oxygen molecules at 0°C is 160 m/s. Find the rms speed of hydrogen molecules at 0°C.

(1) 40 m/s (2) 80 m/s (3) 640 m/s (4) 332 m/s

Ans. (3)

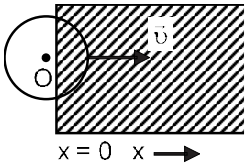
Sol. $V_{rms} \propto \frac{1}{\sqrt{M}}$

$\frac{(V_{rms})O_2}{(V_{rms})H_2} = \sqrt{\frac{m_{H_2}}{m_{O_2}}} = \sqrt{\frac{2}{32}} = \frac{1}{4}$

$(V_{RMS})_{H_2} = 4(V_{RMS})_{O_2} = 4 \times 160 = 640 m/s$

4. A constant magnetic field of 1 T is applied in the x > 0 region. A metallic circular ring of radius 1 m is moving with a constant velocity of 1 m/s along the x-axis. At t = 0 s, the centre O of the ring is at x = – 1m. What will be the value of the induced emf in the ring at t = 1 s?

(Assume the velocity of the ring does not change.)



(1) 0 V (2) 2 V (3) 2π V (4) 1 V

Ans. (1)

Sol. E = B V 2R when half ring enters

E = 1 × 1 × 2 × 1 = 2V

5. A monochromatic neon lamp with wavelength of 670.5 illuminates a photo-sensitive material which has a stopping voltage of 0.48 V. What will be the stopping voltage if the source light is changed with another source of wavelength of 476.6 nm?

(1) 0.24 V (2) 1.5 V (3) 1.25 V (4) 0.96 V

Ans. (3)

Sol. $eV_s = \frac{hc}{\lambda} - \phi$

$$e(V' - V) = hc\left(\frac{1}{\lambda'} - \frac{1}{\lambda}\right)$$

$$V' = \frac{hc}{e}\left(\frac{1}{\lambda'} - \frac{1}{\lambda}\right) + V$$

$$= 1240\left(\frac{1}{474} - \frac{1}{670}\right) + 0.48 = 1.25eV$$

6. The colour coding on a carbon resistor is shown in the given figure. The resistance value of the given resistor is:

(1) (7500 ± 375) Ω (2) (5700 ± 285) Ω (3) (7500 ± 755) Ω (4) (5700 ± 375) Ω

Ans. (1)

Sol. A colour code is used to indicate the resistance value of a carbon resistor and its percentage accuracy.

Colour	Letter as an aid to memory	Number	Multiplier	Colour	Tolerance
Black	B	0	10^0	Gold	5%
Brown	B	1	10^1	Silver	10%
Red	R	2	10^2	No fourth band	20%

Orange	O	3	10^3		
Yellow	Y	4	10^4		
Green	G	5	10^5		
Blue	B	6	10^6		
Violet	V	7	10^7		
Grey	G	8	10^8		
White	W	9	10^9		

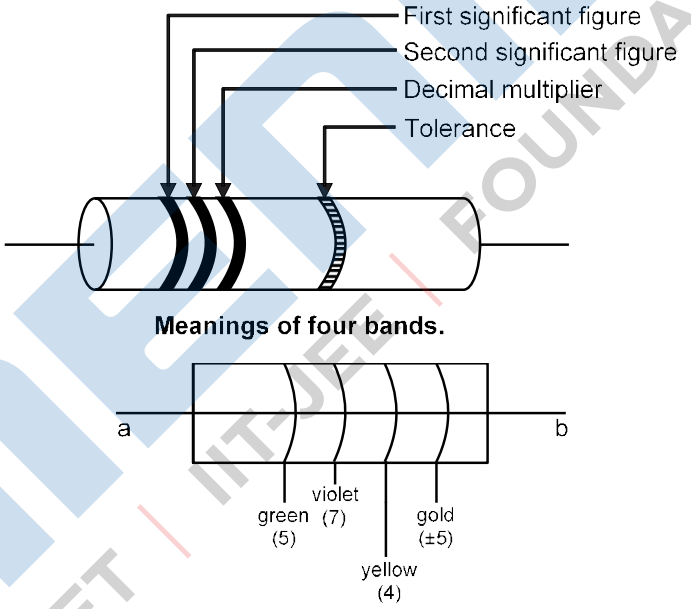
A set of coloured co-axial rings or bands is printed on the resistor which reveals the following facts :

1. The first band indicates the first significant figure.

2. The second band indicates the second significant figure.

3. The third band indicates the power of ten with which the above two significant figures must be multiplied to get the resistance value in ohms.

4. The fourth band indicates the tolerance or possible variation in percent of the indicated value. If the fourth band is absent, it implies a tolerance of ± 20%



Meanings of four bands.

R = (7500 ± 375) Ω

7. A player kicks a football with an initial speed of 25 ms^{-1} at an angle of 45° from the ground. What are the maximum height and the time taken by the football to reach at the highest point during motion?

(Take g = 10 ms^{-2})

(1) h_{max} = 15.625 m T = 1.77 s

(2) h_{max} = 3.54 m T = 0.125 s

(3) h_{max} = 10 m T = 2.5 s

(4) h_{max} = 15.625 m T = 3.54 s

Ans. (1)

Sol. $\theta = 45^\circ$

$$H = \frac{u^2 \sin^2\theta}{2g} = \frac{(25)^2 \times \left(\frac{1}{2}\right)}{2\times 10} = \frac{125}{8}\text{m}$$

and time $t = \frac{T}{2}$

$$= \frac{u\sin\theta}{g} = \frac{25\left(\frac{1}{\sqrt{2}}\right)}{10} = \frac{25}{10\sqrt{2}} = \frac{5}{2\sqrt{2}}\text{s}$$

8. The light waves from two coherent sources have same intensity $I_1 = I_2 = I_0$. In interference pattern the intensity of light at minima is zero. What will be the intensity of light at maxima?

(1) I_0 (2) $2\ I_0$ (3) $4\ I_0$ (4) $5\ I_0$

Ans. (3)

Sol. $I_{minimum} = 0$

$$\left(\sqrt{I_1} - \sqrt{I_2}\right)^2 = 0$$

$$\Rightarrow \quad I_1 = I_2$$

$$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = 4I_1$$

9. Water drops are falling from a nozzle of a shower onto the floor from a height of 9.8 m. The drops fall at a regular interval of time. When the first drop strikes the floor, at that instant, the third drop begins to fall. Locate the position of second drop from the floor when the first drop strikes the floor.

(1) 2.94 m (2) 4.18 m (3) 2.45 m (4) 7.35 m

Ans. (4)

Sol. For first drop.

$$h = \frac{1}{2}g(2n)^2$$

For 2nd drop

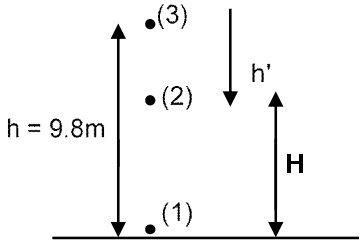
$$h' = \frac{1}{2}g(n)^2$$

$$\frac{h}{h'} = \frac{4}{1}$$

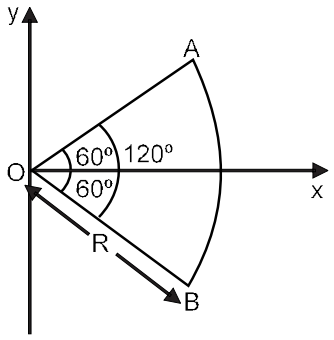
$$h' = \frac{h}{4} = \frac{9.8}{4}$$

so height of 2nd drop

$$sH = h - h' = 9.8 - \frac{9.8}{4} = \frac{3}{4}\times 9.8 = 7.35\text{m}$$



10. Figure shows a rod AB, which is bent in a 120° circular are of radius R. A charge (–Q) is uniformly distributed over rod AB. What is the electric filed $\vec{E}$ at the centre of curvature O?



(1) $\frac{3\sqrt{3}Q}{8\pi^2\varepsilon_0 R^2}(\hat{i})$ (2) $\frac{3\sqrt{3}Q}{8\pi\varepsilon_0 R^2}(\hat{i})$ (3) $\frac{3\sqrt{3}Q}{8\pi^2\varepsilon_0 R^2}(-\hat{i})$ (4) $\frac{3\sqrt{3}Q}{16\pi^2\varepsilon_0 R^2}(\hat{i})$

Ans. (1)

Sol. $\delta = \frac{2k\lambda}{R}\sin\frac{\theta}{2}\hat{i}$

$= \frac{2k}{R}\left(\frac{Q}{2R\frac{\pi}{3}}\right)\sin 60^\circ \hat{i}$

$= \frac{3kQ}{\pi R^2}\frac{\sqrt{3}}{2}\hat{i}$

$= \frac{3\sqrt{3}}{2\pi}\frac{kQ}{R^2}\hat{i}$

11. An antenna is mounted on a 400 m tall building. What will be the wavelength of signal that can be radiated effectively by the transmission tower upto a range of 44 km?

(1) 605 m (2) 302 m (3) 37.8 m (4) 75.6 m

Ans. (1)

Sol. h : height of antenna

λ = wavelength of signal

$\lambda > h$

$\lambda > 400m$

12. Match List-I with List-II.

List – I	**List – II**
(a) R_H (Rydberg constant)	(i) kg m^{-1} s^{-1}
(b) h(Planck's constant)	(ii) kg m^{-2} s^{-1}
(c) μ_B(Magnetic filed energy density	(iii) m^{-1}
(d) η(coefficient of viscosity)	(iv) kg m^{-1} s^{-2}

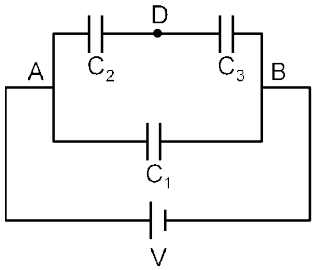
Choose the most appropriate answer from the options given below:

(1) (a) – (iv), (b) – (ii), (c) – (i), (d) – (iii) (2) (a) – (ii), (b) – (iii), (c) – (iv), (d) – (i)

(3) (a) – (iii), (b) – (ii), (c) – (iv), (d) – (i) (4) (a) – (iii), (b) – (ii), (c) – (i), (d) – (iv)

Ans. (3)

13. Three capacitors C_1 = 2 μF, C_2 = 6 μF and C_3 = 12 μF are connected as shown in figure. Find the ratio of the charges on capacitors C_1, C_2 and C_3 respectively



(1) 3 : 4 : 4 (2) 2 : 3 : 3 (3) 2 : 1 : 1 (4) 1 : 2 : 2

Ans. (4)

Sol. $C_{eq} = C_1 + \frac{C_2C_3}{C_2C_3}$

$C_{eq} = 2 + \frac{6\times12}{6+12} = 6F$

then

$\frac{Q_{C_1}}{Q_{C_2}} = \frac{C_1V}{\frac{C_2C_3}{C_2+C_3}V} = \frac{2\times(6+12)}{6\times12} = \frac{1}{2}$

$Q_{C_2} = Q_{C_3}$

14. If force (F), length (L) and time (T) are taken as the fundamental quantities. Then what will be the dimension of density:

(1) $[FL^{-4}T^2]$ (2) $[FL^{-3}T^3]$ (3) $[FL^{-3}T^2]$ (4) $[FL^{-5}T^2]$

Ans. (1)

Sol. $d \propto F^a L^b T^c$

$[M L^{-3}] = [M LT^{-2}]^a [L^b] [T^c]$

$= [M^a L^{a+b} T^{-2a+c}]$

comparing :

$a = 1$ .....(i)

$a + b = -3$ .....(ii)

$-2a + c = 0$ .....(iii)

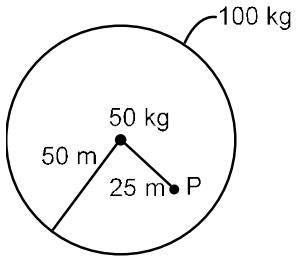
$c = 2$; $b = -4$; $\therefore$ $[d] = [F L^{-4} T^2]$

15. A mass of 50 kg is placed at the centre of a uniform spherical shell of mass 100 kg and radius 50 m. If the gravitational potential at a point, 25 m from the centre is V kg/m. The value of V is:

(1) + 2 G (2) – 20 G (3) – 4 G (4) – 60 G

Ans. (3)

Sol.



$$V_P = -\frac{G(100)}{50} - \frac{G(50)}{25}$$

$= -4G$

16. A coaxial cable consists of an inner wire of radius 'a' surrounded by an outer shell of inner and outer radii 'b' and 'c' respectively. The inner wire carries an electric current i_0, which is distributed uniformly across cross-sectional area. The outer shell carries an equal current in opposite direction and distributed uniformly. What will be the ratio of the magnetic field at a distance x from the axis when (i) $x < a$ and (ii) $a < x < b$?

(1) $\frac{a^2}{x^2}$ (2) $\frac{b^2-a^2}{x^2}$ (3) $\frac{x^2}{b^2-a^2}$ (4) $\frac{x^2}{a^2}$

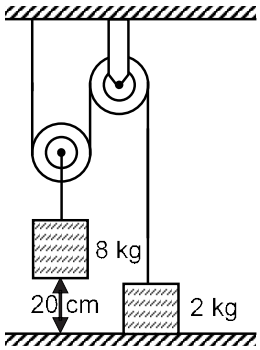
Ans. (4)

Sol. $B_1 \times 2\pi(x) = \frac{\mu_0 I_0 (x)^2}{a^2}$

$B_2 = 2\pi(x) = \mu_0 I_0$

$\frac{B_1}{B_2} = \frac{x^2}{a^2}$

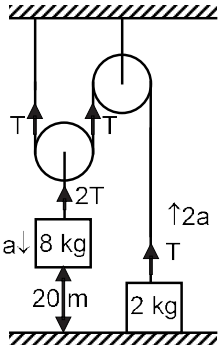
17. The boxes of masses 2 kg and 8 kg are connected by a massless string passing over smooth pulleys. Calculate the time taken by box of mass 8 kg to strike the ground starting from rest. (use $g = 10\ m/s^2$):



(1) 0.25 s (2) 0.34 s (3) 0.2 s (4) 0.4 s

Ans. (4)

Sol.



$8g - 2T = 8a$

$4g = 16a \Rightarrow a = \frac{5}{2} m/s^2$

Thus, $s = ut + \frac{1}{2}at^2$

$0.2 = 0 + \frac{1}{2} \times \frac{5}{2} \times t^2$

$t = 0.4$ s

18. The height of victoria falls is 63 m. What is the difference in temperature of water at the top and at the bottom of fall? [Given 1 cal = 4.2J and specific heat of water = 1 cal g^{-1} °C^{-1}]

(1) 14.76 °C (2) 1.476 °C (3) 0.147 °C (4) 0.014°C

Ans. (3)

Sol. $mgh = ms\Delta\theta$

$g \times 63 = \Delta\theta \times 4.2 \times 10^3$

$\Delta\theta = 0.147°C$

19. Two discs have moments of intertia I_1 and I_2 about their respective axes perpendicular to the plane and passing through the centre. They are rotating with angular speeds, ω_1 and ω_2 respectively and are brought into contact face to face with their axes of rotation coaxial. The loss in kinetic energy of the system in the process is given by:

(1) $\frac{I_1I_2}{(I_1+I_2)}(\omega_1+\omega_2)^2$ (2) $\frac{(\omega_1-\omega_2)^2}{2(I_1+I_2)}$ (3) $\frac{I_1I_2}{2(I_1+I_2)}(\omega_1-\omega_2)^2$ (4) $\frac{(I_1-I_2)^2\omega_1\omega_2}{2(I_1+I_2)}$

Ans. (3)

Sol. Angular Momentum conservation

$I_1\omega_1 - I_2\omega_2 = (I_1 + I_2)\omega$

$\omega = \frac{I_1\omega_1 - I_2\omega_2}{I_1+I_2}$

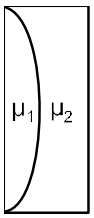
Loss $= \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2 - \frac{1}{2}(I_1+I_2)\omega^2$

$$= \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2 - \frac{1}{2}(I_1 + I_2)\left(\frac{I_1\omega_1 - I_2\omega_2}{I_1 + I_2}\right)^2$$

$$= \frac{1}{2}\frac{I_1I_2}{(I_1 + I_2)}(\omega_1 - \omega_2)^2$$

$$E_i - E_f = \frac{I_1I_2(\omega_1 - \omega_2)^2}{2(I_1 + I_2)}$$

20. Curved surfaces of a plano-convex lens of refractive index μ_1 and a plano-concave lens of refractive index μ_2 have equal radius of curvature as shown in figure. Find the ratio of radius of curvature to the focal length of the combined lenses



(1) $\frac{1}{\mu_2 - \mu_1}$ (2) $\frac{1}{\mu_1 - \mu_2}$ (3) $\mu_2 - \mu_1$ (4) $\mu_1 - \mu_2$

Ans. (4)

Sol. $\frac{1}{f_1} = (\mu_1 - 1)\left[\frac{1}{R}\right]$

$$\frac{1}{f_2} = (\mu_2 - 1)\left[\frac{1}{-R}\right] \Rightarrow \frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{(\mu_1 - 1)}{R} - \frac{(\mu_2 - 1)}{R}$$

$$\frac{1}{f_{eq}} = \frac{(\mu_1 - \mu_2)}{R} \Rightarrow \frac{R}{f_{eq}} = \mu_1 - \mu_2$$

Numeric Value Type

This Section contains **10 Numeric Value Type question**, out of 10 only 5 have to be done.

1. A heat engine operates between a cold reservoir at temperature T_2 = 400 K and a hot reservoir at temperature T_1. It takes 300 J of heat from the hot reservoir and delivers 240 J of heat to the cold reservoir in a cycle. The minimum temperature of the hot reservoir has to be……K.

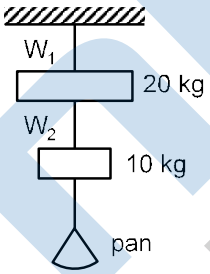
Ans. (500)

Sol. $\eta = \frac{W}{Q_H} = 1 - \frac{T_L}{T_H}$

$\frac{60}{300} = 1 - \frac{400}{T_H}$

T_H = 500 K

2. Wires W_1 and W_2 are made of same material having the breaking stress of 1.25×10^9 N/m^2. W_1 and W_2 have cross-sectional area of 8×10^{-7} m^2 and 4×10^{-7} m^2, respectively. Masses of 20 kg and 10 kg hang from them as shown in the figure. The maximum mass that can be placed in the pan without breaming the wires is_____kg (Use g = 10 m/s^2)



Ans. (40)

Sol. $F1_{max}$ = stress × area

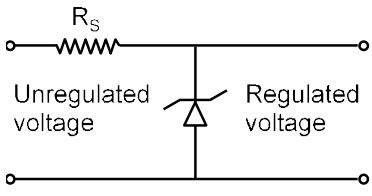
= $1.25 \times 8 \times 10^2$ = 1000 N

$F2_{max}$ = 500 N

$F2_{max}$ = (10 + M) g = 500 N

M = 40 kg

3. A zener diode of power rating 2 W is to be used as a voltage regulator. If the zener diode has a breakdown of 10 V and it has to regulate voltage fluctuated between 6 V and 14 V, the value of R_S for safe operation should be_____Ω.



Ans. (20)

Sol. for diode P = V_i

$$i = \frac{P}{V} = \frac{2}{10} = 0.2mA$$

Now for V_S = 14V

Voltage drop across R_L = 14 – 14 = 4V

Thus, $V = iR_s$

$$R_L = \frac{4}{0.2\times10^{-3}} = 20m\Omega$$

4. A plane electromagnetic wave with frequency of 30 MHz travels in free space. At particular point in space and time, electric field is 6 V/m. The magnetic field at this point will be $x \times 10^{-8}$ T. The value of x is_____.

Ans. (2)

Sol. $$B = \frac{E}{C} = \frac{6}{3\times10^8} = 2\times10^{-8}T$$

5. Two simple harmonic motion, are represented by the equations

$$y_1 = 10\sin\left(3\pi t + \frac{\pi}{3}\right);\ y_2 = 5\left(\sin 3\pi t + \sqrt{3}\cos 3\pi t\right)$$

Ratio of amplitude of y_1 to y_2 = x : 1. The value of x is _____

Ans. (1)

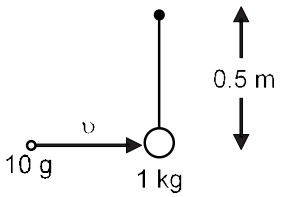
Sol. $$y_1 = 10\sin\left(3\pi t + \frac{\pi}{3}\right)$$

$$y_2 = 5\left[\sin 3\pi t + \sqrt{3}\cos \pi t\right]$$

$$y_2 = 10\sin\left(3\pi t + \frac{\pi}{3}\right) \Rightarrow \frac{A_2}{A_1} = \frac{10}{10} = 1$$

6. A bullet of 10 g, moving with velocity v, collides head-on with the stationary bob of a pendulum and recoils with velocity 100 m/s. The length of the pendulum is 0.5 m and mass of the bob is 1 kg. The minimum value of v = _____m/s so that the pendulum describes a circle.

(Assume the string to be inextensible and g = 10 m/s^2)



Ans. (400)

Sol. Final velocity of system (bullet + mass) should be $= \sqrt{5gR}$

Now $M = \sqrt{5gR} = m(v + 100)$

thus $v = \frac{M}{m}\sqrt{5gr} - 100 = 500 - 100 = 400 m/s$

v = 400 m/s

7. A tuning fork is vibrating at 250 Hz. The length of the shortest closed organ pipe that will resonate with the tuning fork will be_____cm. (Take speed of sound in air as 340 ms^{-1})

Ans. (34)

Sol. For closed organ pipe

$$F_{-0} = (2n+1)\frac{v}{4\ell}$$

for minimum length, n = 0

$$f_0 = \frac{v}{4\ell}$$

$$\Rightarrow \quad \ell = \frac{v}{4f_0}$$

$$= \frac{340}{4\times 250} = 34cm$$

8. An ac circuit has an inductor and a resistor of resistance R in series, such that X_L = 3R. Now, a capacitor is added in series such that X_C = 2R. the ratio of new power factor with the old power factor of the circuit is $\sqrt{5} : x$. The value of x is _____

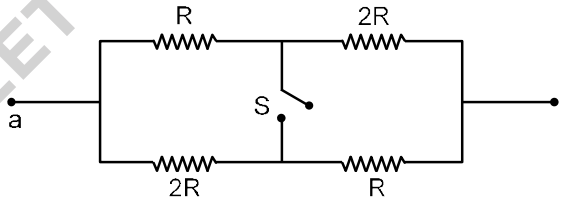
Ans. (1)

Sol. $$PF_{initial} = \frac{R}{\sqrt{9R^2 + R^2}} = \frac{1}{\sqrt{10}}$$

$$PF_{final} = \frac{R}{\sqrt{(3-2)^2R^2 + R^2}} = \frac{1}{\sqrt{2}}$$

$$\frac{PF_{final}}{PF_{initial}} = \sqrt{5}$$

9. The ratio of the equivalent resistance of the network (shown in figure) between the points a and b when switch is open and switch is closed is x : 8. The value of x is_____



Ans. (9)

Sol. When switch is closed

$$R_1 = \frac{2R}{3} + \frac{2R}{3} = \frac{4R}{3}$$

When open $R_2 = \frac{3R \cdot 3R}{3R + 3R} = \frac{3R}{2}$

$\frac{R_1}{R_2} = \frac{4R}{3} \times \frac{2}{3R} = \frac{8}{9} = \frac{8}{x}$

10. X different wavelength may be observed in the spectrum from a hydrogen sample is the atoms are exited to states with principle quantum number n = 6 ? The value of X is _____

Ans. (15)

Sol. $6C_2 = 15$

PART B : CHEMISTRY

Single Choice Type

This section contains **20 Single choice questions**. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

1. The oxide that gives H_2O_2 most readily on treatment with H_2O is :

(1) Na_2O_2 (2) $BaO_2 \cdot 8H_2O$ (3) SnO_2 (4) PbO_2

Ans. (1)

Sol. (i) $Na_2O_2 + H_2O \rightarrow NaOH + H_2O_2$ (ii) $Li_2O + H_2O \rightarrow 2LiOH$

(iii) $MgO + H_2O \rightarrow Mg(OH)_2$ (iv) $KO_3 + H_2O \rightarrow KOH + O_2$

2. The addition of dilute NaOH to Cr^{3+} salt solution will give :

(1) a solution of $[Cr(OH)_4]^-$ (2) precipitate of $[Cr(OH)_6]^{3-}$

(3) precipitate of $Cr_2O_3(H_2O)_n$ (4) precipitate of $Cr(OH)_3$

Ans. (3)

Sol. $Cr^{3+} \xrightarrow{NaOH} \underset{\text{(Green ppt.)}}{Cr(OH)_3 \downarrow}$

3. The correct order of ionic radii for the ions, P^{3-}, S^{2-}, Ca^{2+}, K^+, Cl^- is :

(1) $P^{3-} > S^2 > Cl^- > K^+ > Ca^{2+}$ (2) $P^{3-} > S^2 > Cl^- > Ca^{2+} > K^+$

(3) $Cl^- > S^2 > P^{3-} > Ca^{2+} > K^+$ (4) $K^+ > Ca^{2+} > P^{3-} > S^{2-} > Cl^-$

Ans. (1)

Sol.

	P^{3-}	S^{2-}	Cl^-	K^+	Ca^{2+}
Z	15	16	17	19	20
No.of e^-	18	18	18	18	18

For isoelectronic species greater is Z smaller is radius. So correct order

$P^{3-} > S^{2-} > Cl^- > K^+ > Ca^{2+}$

4. Choose the correct statement from the following :

(1) LiF has least negative standard enthalpy of formation among alkali metal fluorides.

(2) The standard enthalpy of formation for alkali metal bromides becomes less negative on descending the group.

(3) The low solubility of CsI in water is due to its high lattice enthalpy.

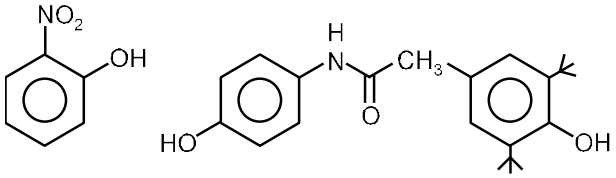
(4) Among the alkali metal halides, LiF is least soluble in water.

Ans. (4)

Sol. (3) CsI has less lattice enthalpy. But due to less hydration enthalpy of Cs^+ ion it is less soluble.

(4) LiF is least soluble in water.

5. The compound/s which will show significant intermolecular H-bonding is/are :



(a) (b) (c)

(1) (a), (b) and (c) (2) (b) only (3) (c) only (4) (a) and (b) only

Ans. (2)

Sol. (a) Shows intra molecular H-bonding.

(b) Shows significant intermolecular H-bonding.

(c) It do not show intermolecular H-bonding due to steric hindrance

6. In stratosphere most of the ozone formation is assisted by :

(1) γ-rays (2) cosmic rays (3) visible radiations (4) ultraviolet radiation

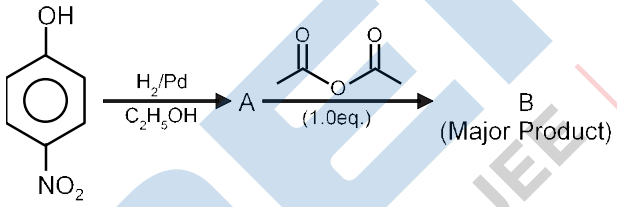
Ans. (4)

Sol. Ozone in the stratosphere is a product of UV radiations acting on dioxygen (O_2) molecules.

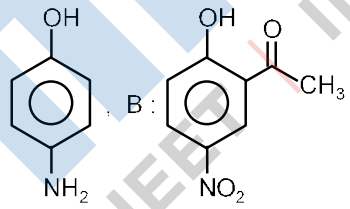
$$O_2(g) \xrightarrow{UV} O(g) + O(g)$$

$$O(g) + O_2(g) \underset{}{\overset{UV}{\rightleftharpoons}} O_3(g)$$

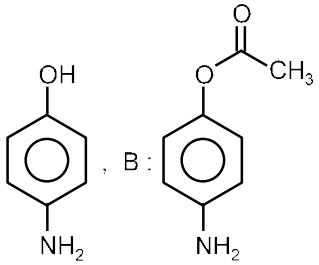
7. The correct structures of A and B formed in the following reactions are :



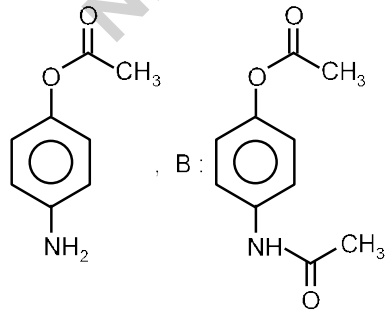
(1) A : , B :



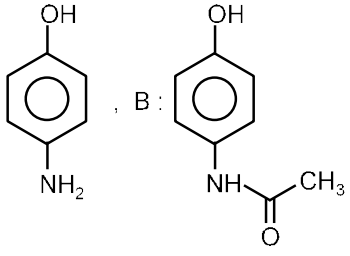
(2) A : , B :



(3) A : , B :

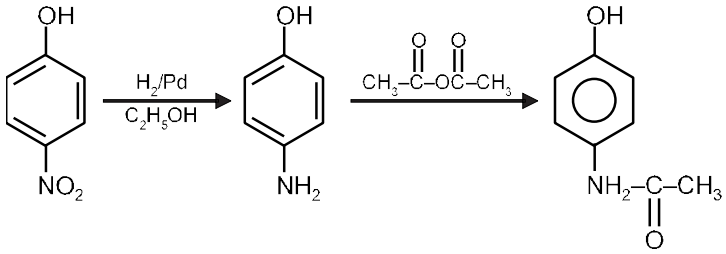


(4) A : , B :



Ans. (4)

Sol.



8. Lyophilic sols are more stable than lyophobic sols because,

(1) the colloidal particles have no charge.

(2) the colloidal particles have positive charge.

(3) the colloidal particles are solvated.

(4) there is a strong electrostatic repulsion between the negatively charged colloidal particles.

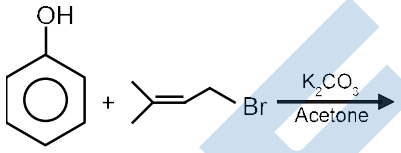
Ans. (3)

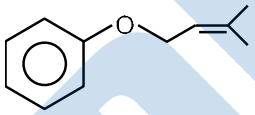
Sol. Lyophilic sols are quite stable and cannot be easily coagulated.

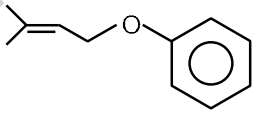
Lyophobic sols need stabilisingagents for their preservation.

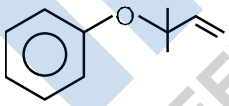
Property	Lyophilic	Lyophobic colloids
Stability	These are very stable	These are unstable and require traces of stabilizers

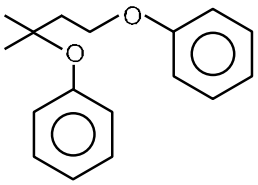
9. The Crystal Stabilization Energy (CFSE) and magnetic moment (spin-only) of an octahedral aqua complex of a metal ion (MZ^{+}) are $-0.8\ \Delta_0$ and 3.87 BM, respectively. Identify (M^{Z+}):



(1) 

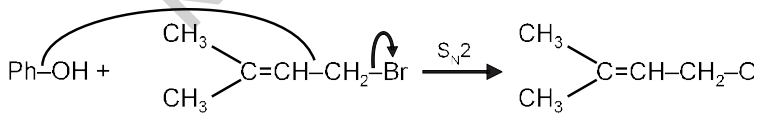
(2) 

(3) 

(4) 

Ans. (1)

Sol.



10. Which one of the following tests used for the identification of functional groups in organic compounds dues not use copper reagent ?

(1) Seliwanoff's test

(2) Biuret test for peptide bond

(3) Barfoed's Test

(4) Benedict's test

Ans. (1)

Sol. **Seliwanoff Test :** This test use to detect the presence of Ketohexose than aldohexose.

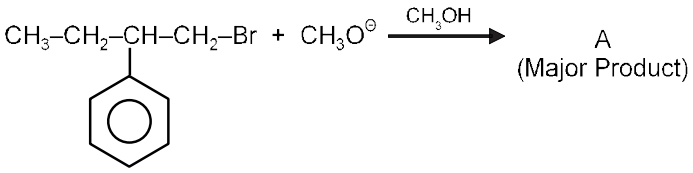
Reagent : Resorcinol + conc. HCl.

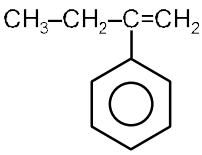
Barfoed Test : It if used to detect Monosaccharide by Reduction of Cu(II) into Copper (I).

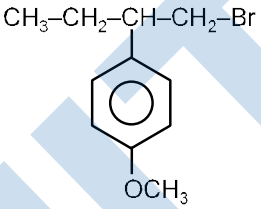
Fehling Solution : It is used to defect aldehyde than Ketone by reduction Cu(II) into Cu(I).

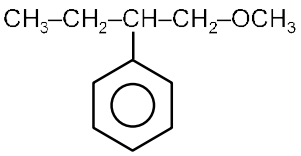
Biurate Test : Use to detect presence of peptide bond. Protein by reduction of Cu(II) into Cu(I).

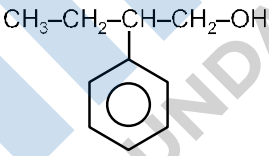
11. The major product (A) formed in the reaction given below is :



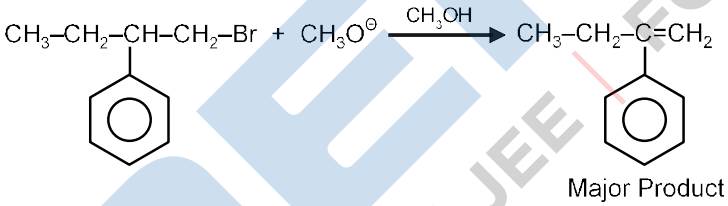
(1) 

(2) 

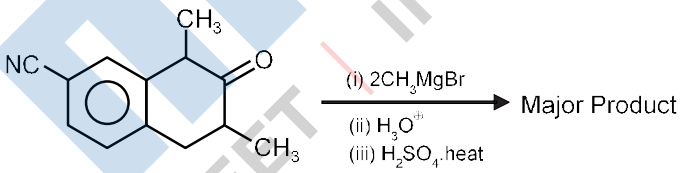
(3) 

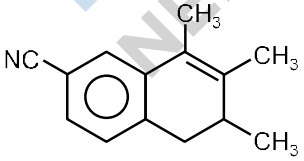
(4) 

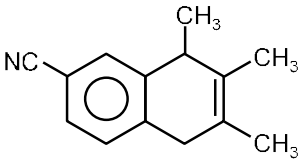
Ans. (1)

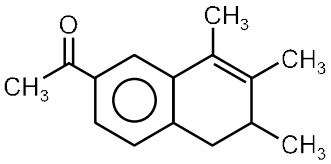
Sol. 

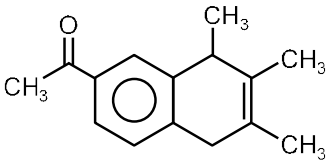
12. Which one of the following is the major product of the given reaction ?



(1) 

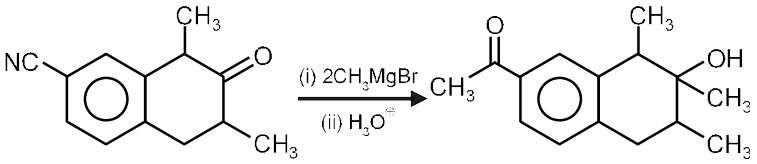
(2) 

(3) 

(4) 

Ans. (3)

Sol.



13. Match List-I with List-II :

List-I **(Name of ore/mineral)**	**List-II** **(Chemical formula)**
(a) Calamine	(i) ZnS
(b) Malechite	(ii) $FeCO_3$
(c) Siderite	(iii) $ZnCO_3$
(d) Sphalerite	(iv) $CuCO_3.Cu(OH)_2$

Choose the most appropriate answer from the options given below :

(1) (a) – (iv), (b) – (iii), (c) – (i), (d) - (ii)

(2) (a) – (iii), (b) – (ii), (c) – (iv), (d) - (i)

(3) (a) – (iii), (b) – (iv), (c) – (i), (d) - (ii)

(4) (a) – (iii), (b) – (iv), (c) – (ii), (d) - (i)

Ans. (4)

Sol. (i) Calamine : $ZnCO_3$

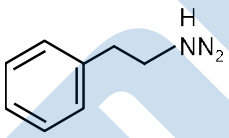
(ii) Siderite : $FeCO_3$

(iii) Malechite : $CuCO_3.Cu(OH)_2$

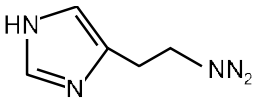
(iv) Sphalerite : ZnS

14. Which one of the following chemicals is responsible for the production of HCl in the stomach leading to irritation and pain ?

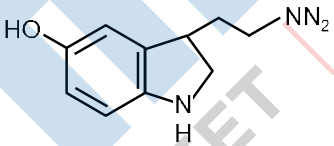
(1)



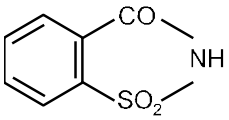
(2)



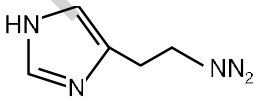
(3)



(4)



Ans. (2)

Sol. Histamine  is responsible for acidity in stomach and stomach disorder.

15. Which one of the following is formed (mainly) when red phosphorus is heated in a sealed tube at 803 K?

(1) α-Black phosphorus

(2) White phosphorus

(3) Yellow phosphorus

(4) β-Black phosphorus

Ans. (1)

Sol. When red phosphorous is heated in a sealed tube at 800 K, then α-black phosphorous is formed.

16. Hydrolysis of sucrose gives :

(1) α-D-(+)-Glucose and α-D-(–)-Fructose (2) α-D-(+)-Glucose and β-D-(–)-Fructose

(3) α-D-(–)-Glucose and β-D-(–)-Fructose (4) α-D-(–)-Glucose and α-D-(+)-Fructose

Ans. (2)

Sol. Sucrose is a disaccharide of α-Glucose and β-Fructose. So on hydrolysis we obtain α-Glucose and β-Fructose.

17. Potassium permanganate on heating at 513 K gives a product which is :

(1) paramagnetic and colourless (2) diamagnetic and colourless

(3) diamagnetic and green (4) paramagnetic and green

Ans. (4)

Sol. $\underset{\text{Purple}\atop\text{(Diamagnetic)}}{2KMnO_4} \longrightarrow \underset{\text{Green}\atop\text{(Paramagnetic)}}{K_2\overset{+6}{M}nO_4} + \overset{+4}{M}nO_2(s) + O_2$

18. Which one of the following reactions will not yield propionic acid ?

(1) $CH_3CH_2CH_2Br$ + Mg, CO_2 dry ether/H_3O^+ (2) $CH_3CH_2CH_3$ + $KMnO_4$(Heat), OH^- / H_3O^+

(3) $CH_3CH_2COCH_3$ + OH^-/ H_3O^+ (4) $CH_3CH_2COOH_3$ + OI^-/ H_3O^+

Ans. (1)

Sol. $CH_3CH_2CH_2Br \xrightarrow{Mg} CH_3CH_2CH_2MgBr \xrightarrow{CO_2/H_3O^+} \underset{\text{Butanoic acid}}{CH_3CH_2CH_2COOH}$

19. Given below are two statements :

Statement I : Ethyl pent-4-yn-oate on reaction with CH_3MgBr gives a 3°-alcohol.

Statement II : In this reaction one mole of ethyl pent-4-yn-oate utilizes two moles of CH_3MgBr.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both Statement I and Statement II are false (2) Statement I is true but Statement II is false

(3) Statement I is false but Statement II is true (4) Both Statement I and Statement II are true

Ans. (2)

Sol. $HC \equiv C - C - C - \overset{\overset{O}{\|}}{C} - OEt$ utilizes 3 moles of CH_3MgBr. One mole for acidic H of alkyne & 2 moles for ester group.

20. Which one of the following is used to remove most of plutonium from spent nuclear fuel ?

(1) BrO_3 (2) I_2O_5 (3) ClF_3 (4) O_2F_2

Ans. (4)

Sol. O_2F_2 oxidise plutonium to PuF_6 and the reaction is used to removing plutonium as PuF_6 from spent nuclear fuel.

Numeric Value Type

This Section contains **10 Numeric Value Type question**, out of 10 only 5 have to be done.

1. The number of photons emitted by a monochromatic (single frequency) infrared range finder of power 1 mW and wavelength of 1000 nm, in 0.1 second is $x \times 10^{13}$. The value of x is _____.

($h = 6.63 \times 10^{-34}$ Js, $c = 3.00 \times 108$ ms^{-1})

Ans. (50)

Sol. Energy emitted in 0.1 sec

$= 0.1\text{sec.} \times 10^{-3} \frac{J}{s}$

$= 10^{-4}$ J

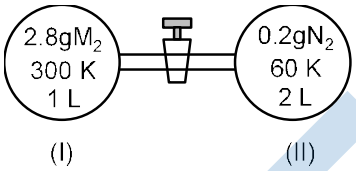
If 'n' photons of λ = 1000 nm are emitted, then $10^{-4} = n = \frac{hc}{\lambda}$

$\Rightarrow 10^{-4} = \frac{n \times 6.63 \times 10^{-34} \times 310^8}{1000 \times 10^{-9}}$

$\Rightarrow n = 5.02 \times 10^{14} = 50.2 \times 10^{13}$

$\Rightarrow 50$

2. Two flasks I and II shown below are connected by a value of negligible volume.



When the value, the final pressure of the system in bar is $x \times 10^{-2}$. The value of x is _____.

[**Assume :** Ideal gas; 1 bar = 10^5 Pa : Molar mass of N_2 = 28.0 mol^{-1}; R = 8.31 J mol^{-1} K^{-1}]

Ans. (84)

Sol. Applying ; $(n_1 + n_{II})_{initial} = (n_1 + n_{11})_{final}$

$\Rightarrow$ Assuming the system attains a final temperature of T (such that 300 < T < 60)

$\Rightarrow$ {|Heat lost by N_2 of container (I)| = |Heat gained by N_2 of container II|}

$\Rightarrow$ $n_1 C_m (300 - T = n_{II} C_m (T - 60))$

$\Rightarrow$ $\left(\frac{2.8}{28}\right)(30 - T) = \frac{0.2}{28}(T - 60)$

$\Rightarrow$ $14(300 - T) = T - 60$

$\Rightarrow$ T = 284 K (final temperature)

$\Rightarrow$ If the final pressure = P

$\Rightarrow$ $(n_1 + n_{II})_{final}\left(\frac{3.0}{28}\right)$

$$\Rightarrow \frac{P}{RT}(V_I + V_{II}) = \frac{3.0gm}{28gm/mol}$$

$$P = \frac{3}{28}mol \times 8.31\frac{J}{mol-K} \times \frac{284K}{3\times10^{-3}m^3} \times 10^5 \frac{bar}{Pa}$$

$\Rightarrow$ 0.84287 bar

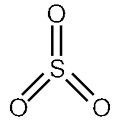
$\Rightarrow$ 84.28 × 10^{-2} bar

$\Rightarrow$ 84

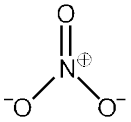
3 The number of species having non-pyramidal shape among the following is _____.

Ans. (3)

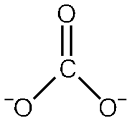
Sol.



Trigonal planar



Trigonal planar



Trigonal planar

Pyr.

Hence non-pyramidal species are SO_3, NO_3^- and CO_3^{2-}.

4. The first order rate constant for the composition of $CaCO_3$ at 700 K is 6.36 × 10^{-3} s^{-1} and activation energy is 209 kJ mol^{-1}. Its rate constant (in s^{-1}) at 600 K is x × 10^{-6}. The value of x is _____.

[Given R = 8.31 J K^{-1} mol^{-1} ; log 6.36 × 10^{-3} = –2.19, $10^{-4.79}$ = 1.62 × 10^{-5}]

Ans. (16)

Sol. K_{700} = 6.36 × 10^{-3} s^{-1} ;

K_{600} = x × 10^{-3} s^{-1}

Applying ;

$$\log\left(\frac{K_{T_2}}{K_{T_1}}\right) = \frac{-E_a}{2.303R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

$$\log\left(\frac{K_{700}}{K_{600}}\right) = \frac{-E_a}{2.303R}\left(\frac{1}{700} - \frac{1}{600}\right)$$

$$\log\left(\frac{6.36\times10^{-3}}{K_{600}}\right) = \frac{+209\times1000}{2.303\times8.31}\left(\frac{100}{700\times600}\right)$$

$\log(6.36 \times 10^{-3}) - \log K_{600}$ = 2.6

$\Rightarrow \log K_{600}$ = –2.19 – 2.6 = –4.79

$\Rightarrow K_{600} = 10^{-4.79} = 1.62 \times 10^{-5}$

$= 16.2 \times 10^{-6}$

$= x \times 10^{-6}$

$\Rightarrow$ x = 16

5. 100 g of propane is completely reacted with 1000 g of oxygen. The mole fraction of carbon dioxide in the resulting mixture is $x \times 10^{-2}$. The value of x is _____ .

Ans. (19)

Sol. $C_3H_8(g) + 5O_2(g) \longrightarrow 3CO_2(g) + 4H_2O(\ell)$

t = 0	2.27 mole	31.25 mol		
t = ∞	0	19.9 mol	6.81 mol	9.08 mol

mole fraction of CO_2 in the final reaction mixture (heterogenous)

$$X_{CO_2} = \frac{6.81}{19.9 + 6.81 - 9.08}$$

$= 0.1902 = 1902 \times 10^{-2} \Rightarrow 19$

6. The resistance of conductivity cell with cell constant 1.14 cm^{-1}, containing 0.001 M KCl at 298 K is 1500Ω. The molar conductivity of 0.001 M KCl solution at 298 K in S cm^2 mol^{-1} is _____.

[Atomic masses : Cu : 63.54 u, S: 32 u, O : 16 u, H : 1 u]

Ans. (760)

Sol. $$K = \frac{1}{R} \times \frac{\ell}{A} = \left(\left(\frac{1}{1500}\right) \times 1.14\right) Scm^{-1}$$

$$\Rightarrow \wedge_m = 1000 \times \frac{\left(\frac{1.14}{1500}\right)}{0.001} Scm^2mol^{-1}$$

$= 760$ S cm^2 $mol^{-1} \Rightarrow 760$

7. When 5.1 g of solid NH_4HS is introduced into a two litre evacuated flask at 27°C, 20% of the solid decomposes into gaseous ammonia and hydrogen sulphide. The K_p for the reaction at 27°C is $x \times 10^{-2}$. The value of x is _____ .

[Given R = 0.082 L atm K^{-1} mol^{-1}]

Ans. (6)

Sol. Moles of NH_4HS initially taken $= \frac{5.1g}{51g/mol} = 0.1mol$

volume of vessel = 2ℓ

	$NH_4HS(s)$	$\rightleftharpoons$	$NH_3(g)$	+	$H_2S(g)$
t = 0	0.1 mol				
t = ∞	0.1 (1–0.2)		0.1×0.2		0.1×0.2

$\Rightarrow$ partial pressure of each component

$$P = \frac{nRT}{V} = \frac{0.1 \times 0.2 \times 0.082 \times 300}{2}$$

$= 0.246$ atm

$$\Rightarrow k_P = P_{NH_3} \times P_{H_2S} = (0.246)^2 = 0.060516$$

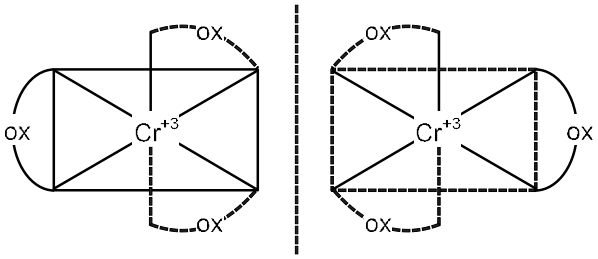
$= 6.05 \times 10^{-2}$

$\Rightarrow 6$

8. The number of optical isomers possible for $[Cr(C_2O_4)_3]^{3-}$ is _____.

Ans. (2)

Sol. The number of optical isomers for $[Cr(C_2O_4)_3]^{3-}$ is two.



9. 40 g of glucose (Molar mass = 180) is mixed with 200 mL of water. The freezing point of solution is ____ K.

[Given : K_f = 1.86 K kg mol^{-1}; Density of water = 1.00 g cm^{-3} ; Freezing point of water = 273.15 K]

Ans. (271)

Sol. $\text{Molality} = \frac{\left(\frac{40}{180}\right)\text{mol}}{0.2\text{Kg}} = \left(\frac{10}{9}\right)\text{molal}$

$\Rightarrow \Delta T_f = T_f - T'_f = 1.86 \times \frac{10}{9}$

$\Rightarrow T'_f = 273.15 - 1.86 \times \frac{10}{9}$

$= 271.08$ K

≈ 271 K

10. Data given for the following reaction is as follows :

$$FeO(s) + C_{(graphite)} \longrightarrow Fe(s) + CO(g)$$

Substance	$\Delta_f H^\circ$	ΔS° (J mol^{-1} K^{-1})
FeO(s)	–266.3	57.49
$C_{(graphite)}$	0	5.74
Fe(s)	0	27.28
CO(g)	–110.5	197.5

Ans. (964)

Sol. $T_{min} = \left(\frac{\Delta^\circ H}{\Delta^\circ S}\right)$

$\Delta^\circ H_{rxn} = \left[\Delta_f^0 H(Fe) + \Delta_f^0 H(CO)\right] -$

= 155.8 kJ/mol

$\Delta^\circ S_{rxn}$ = [$\Delta^\circ S(Fe)$ + $\Delta^\circ S(CO)$] – [$\Delta^\circ S(FeO)$ + $\Delta^\circ S(C_{graphite})$]

= [27.28 + 197.6] – 57.49 + 5.74

= 161.65 J/mol–K

$$T_{min} = \frac{155.8 \times 10^3 J/mol}{161.65 J/mol - K} = 963.8K$$

≈ 964 K

PART C : MATHEMATICS

Single Choice Type

This section contains **20 Single choice questions**. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

1. The value of the integral $\int_0^1 \frac{\sqrt{x}dx}{(1+x)(1+3x)(3+x)}$ is :

(1) $\frac{\pi}{4}\left(1-\frac{\sqrt{3}}{2}\right)$ (2) $\frac{\pi}{8}\left(1-\frac{\sqrt{3}}{6}\right)$ (3) $\frac{\pi}{8}\left(1-\frac{\sqrt{3}}{2}\right)$ (4) $\frac{\pi}{4}\left(1-\frac{\sqrt{3}}{6}\right)$

Ans. (3)

Sol. Let $\sqrt{x}=t$

$$=\int_0^1 \frac{2t^2dt}{(t^2+1)(3t^2+1)(t^2+3)}=\int_0^1 \frac{((3t^2+1)-(t^2+1))dt}{(t^2+1)(3t^2+1)(t^2+3)}=$$

$$=\int_0^1\left(\frac{1}{(t^2+3)(t^2+1)}-\frac{1}{(t^2+3)(3t^2+1)}\right)dt$$

$$=\int_0^1\left(\frac{dt}{2(t^2+1)}-\frac{1}{8}\frac{3dt}{(3t^2+1)}-\frac{3}{8}\frac{dt}{(t^2+3)}\right)$$

$$=\left(\frac{1}{2}\tan^{-1}t-\frac{3\sqrt{3}}{8\times 3}\tan^{-1}\sqrt{3}t-\frac{3}{8\sqrt{3}}\tan^{-1}\frac{t}{\sqrt{3}}\right)_0^1$$

$$=\frac{\pi}{8}-\frac{\sqrt{3}}{8}\times\frac{\pi}{3}-\frac{\sqrt{3}}{8}\frac{\pi}{6}=\frac{\pi}{8}-\frac{\sqrt{3}\pi}{16}$$

2. The Boolean expression $(p\wedge q)\Rightarrow((r\wedge q)\wedge p)$ is equivalent to :

(1) $(p\wedge q)\Rightarrow(p\wedge q)$ (2) $(p\wedge r)\Rightarrow(p\wedge q)$

(3) $(p\wedge q)\Rightarrow(r\wedge q)$ (4) $(p\wedge q)\Rightarrow(r\vee q)$

Ans. (3)

Sol. $(p\wedge q)\Rightarrow((r\wedge q)\wedge p)\equiv(p\wedge q)\Rightarrow(r\wedge(p\wedge q))$ ....(i)

Case I : when $p\wedge q$ = T and r = F

Statement-(i) is false as well as option (3) is also false option (1),

option (2) are True so given expression is not equivalent to option (1), option (2) option (4) may be true if we consider q = T $\therefore$ option (4) is also not equivalent to option (1)

Case II : $p\wedge q$ = False then statement (i) is true as well as option (3) is true.

by both cases we can say option (3) is equivalent to given statement (1)

3. A box open from top is made from a rectangular sheet dimension a × b by cutting squares each of side x from each of the four corners and folding up the flaps. If the volume of the box is maximum, then x is equal to :

(1) $\frac{a+b+\sqrt{a^2+b^2-ab}}{6}$ (2) $\frac{a+b-\sqrt{a^2+b^2-ab}}{12}$

(3) $\frac{a+b-\sqrt{a^2+b^2-ab}}{6}$ (4) $\frac{a+b-\sqrt{a^2+b^2+ab}}{6}$

Ans. (3)

Sol. $v(x) = (a - 2x)(b - 2x)x$

$v'(x) = 12x^2 - 4(a + b)x + ab = 0$

for max $x = \frac{a+b-\sqrt{a^2+b^2-ab}}{6}$

4. If $y(x)=\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right), x\in\left(\frac{\pi}{2},\pi\right)$, then $\frac{dy}{dx}$ at $x=\frac{5\pi}{6}$ is :

(1) 0 (2) –1 (3) $\frac{-1}{2}$ (4) $\frac{1}{2}$

Ans. (3)

Sol. $y(x)=\cot^{-1}\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}$

$$y(x)=\cot^{-1}\frac{\left|\cos\frac{x}{2}+\sin\frac{x}{2}\right|+\left|\cos\frac{x}{2}-\sin\frac{x}{2}\right|}{\left|\cos\frac{x}{2}+\sin\frac{x}{2}\right|-\left|\cos\frac{x}{2}-\sin\frac{x}{2}\right|}$$

$$=\cot^{-1}\frac{\cos\frac{x}{2}+\sin\frac{x}{2}+\sin\frac{x}{2}-\cos\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}-\sin\frac{x}{2}+\cos\frac{x}{2}} \text{ for } x\in\left(\frac{\pi}{2},\pi\right)$$

$$=\cot^{-1}\left(\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}\right)=\cot^{-1}\left(\tan\frac{x}{2}\right)$$

$y(x)=\frac{\pi}{2}-\frac{x}{2}$

$y'(x)=-\frac{1}{2}$

5. If $\lim\limits_{x\to\infty}\left(\sqrt{x^2-x+1}-ax\right)=b$, then the ordered pair (a, b) is :

(1) $\left(1,-\frac{1}{2}\right)$ (2) $\left(-1,\frac{1}{2}\right)$ (3) $\left(-1,-\frac{1}{2}\right)$ (4) $\left(1,\frac{1}{2}\right)$

Ans. (1)

Sol. $\lim\limits_{x\to\infty}\left(\sqrt{x^2-x+1}-ax\right)=b$

$\Rightarrow \lim\limits_{x\to\infty}\left(\sqrt{x^2-x+1}-ax\right)\times\dfrac{\sqrt{x^2-x+1}+ax}{\sqrt{x^2-x+1}+ax}=b$

$\Rightarrow \lim\limits_{x\to\infty}\dfrac{(x^2-x+1)-(ax)^2}{\sqrt{x^2-x+1}+ax}=b$

$\Rightarrow$ limit exist only if $a^2 = 1$

$\therefore$ $a = 1, -1$

$\Rightarrow \lim\limits_{x\to\infty}\dfrac{-x+1}{\sqrt{x^2-x+1}+ax}=b$

$\Rightarrow \lim\limits_{x\to\infty}\dfrac{-1+\frac{1}{x}}{\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}+a}=b$

$\Rightarrow \dfrac{-1}{1+a}=b$

But $a \neq -1$,

So $a = 1$

$b=\dfrac{-1}{2}$

$(a,b)\equiv\left(1,\dfrac{-1}{2}\right)$

6. Two poles AB of length a metres and CD of length a + b($b \neq a$) metres are erected at the same horizontal level with bases at B and D. If BD = x and $\tan\angle ACB=\dfrac{1}{2}$, then :

(1) $x^2 + 2(a + 2b)\, x - b(a + b) = 0$

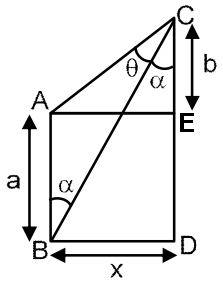
(2) $x^2 + 2(a + 2b)\, x + a(a + b) = 0$

(3) $x^2 - 2ax + b(a + b) = 0$

(4) $x^2 - 2ax + a(a + b) = 0$

Ans. (3)

Sol. From diagram $\tan(\theta+\alpha)=\dfrac{x}{b},\tan\alpha=\dfrac{x}{a+b}$



$\Rightarrow \dfrac{\frac{1}{2}+\frac{x}{a+b}}{1-\frac{1}{2}\cdot\frac{x}{a+b}}=\dfrac{x}{b}\Rightarrow\dfrac{a+b+2x}{2(a+b)-x}=\dfrac{x}{b}$

$\Rightarrow ab + b^2 + 2bx = 2ax + 2bx - x^2$

$\Rightarrow x^2 - 2ax + ab + b^2 = 0$

7. Let Z be the set of all integers,

$A = \{(x, y) \in Z \times Z : (x-2)^2 + y^2 \le 4\}$

$B = \{(x, y) \in Z \times Z : x^2 + y^2 \le 4\}$ and

$C = \{(x, y) \in Z \times Z : (x-2)^2 + (y-2)^2 \le 4\}$

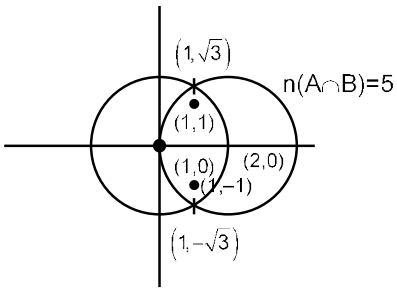
(1) 25 (2) 9 (3) 16 (4) 49

Ans. (1)

Sol. $A = \{(x, y) \in Z \times Z ; (x-2)^2 + y^2 \le 4\}$

$B = \{(x, y) \in Z \times Z ; x^2 + y^2 \le 4\}$

$C = \{(x, y) \in Z \times Z ; (x-2)^2 + (y-2)^2 \le 4\}$



Similarly $n(A \cap B) = 5$

Relation from $A \cap B$ to $A \cap C = 2^{5\times5} = 2^P \Rightarrow p = 25$

8. The equation of the plane passing through the line of intersection of the planes

$\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)=1$ and $\vec{r}.\left(2\hat{i}+3\hat{j}-\hat{k}\right)+4=0$ and parallel to the x-axis is :

(1) $\vec{r}.\left(\hat{i}+3\hat{k}\right)+6=0$ (2) $\vec{r}.\left(\hat{i}-3\hat{k}\right)+6=0$ (3) $\vec{r}.\left(\hat{j}-3\hat{k}\right)-6=0$ (4) $\vec{r}.\left(\hat{j}-3\hat{k}\right)+6=0$

Ans. (4)

Sol. Equation of phase

$(1 + 2\lambda)x + (1 + 3\lambda)y + (1 - \lambda)z = 1 - 4\lambda$ .....(1)

Parallel to x-axis

$(1 + 2\lambda)1 + (1 + 3\lambda)0 + (1 - \lambda)0 = 0$

$\left(1-\frac{3}{2}\right)y+\left(1+\frac{1}{2}\right)z=1+2$

$3z - y = 6$

$\vec{r}.\left(\hat{j}-3\hat{k}\right)+6=0$

9. If the solution curve of the differential equation $(2x - 10y^3)\,dy + ydx = 0$, passes through the points (0,1) and (2, β), then β is a root of the equation?

(1) $y^5 - 2y - 2 = 0$ (2) $y^5 - y^2 - 1 = 0$ (3) $2y^5 - y^2 - 2 = 0$ (4) $2y^5 - 2y - 1 = 0$

Ans. (2)

Sol. $(2x - 10y^3)\,dy\;\; y\,dx = 0$

$$\Rightarrow \frac{dx}{dy} + \frac{2x}{y} = 10y^2$$

$$\Rightarrow \text{I.F.} = e^{2\int \frac{1}{y}dx} = y^2$$

$$\Rightarrow x.y^2 = \int 10y^4 dy$$

$$\Rightarrow x.y^2 = 2y^5 + c$$

$\Rightarrow x = 0 \Rightarrow y = 1\ c = -2$

$$\Rightarrow x.y^2 = 2y^5 - 2$$

Passing Through $(2, \beta)$

$$\Rightarrow 2.\beta^2 = 2\beta^5 - 2$$

$$\Rightarrow \beta^5 - \beta^2 - 1 = 0$$

root of the equations

$$y^5 - y^2 - 1 = 0$$

10. The set of all values of $k > -1$, for which the equation $(3x^2 + 4x + 3)^2 - (k + 1)(3x^2 + 4x + 3)(3x^2 + 4x + 2) + k(3x^2 + 4x + 2)^2 = 0$ has real roots, is :

(1) $\left[-\frac{1}{2}, 1\right)$ (2) $\left(1, \frac{5}{2}\right]$ (3) $\left(\frac{1}{2}, \frac{3}{2}\right] - \{1\}$ (4) [2, 3)

Ans. (2)

Sol. Let $t = (3x^2 + 4x + 2)$

$$\Rightarrow (t + 1)^2 - (K + 1)(t)(t + 1) + K(t)^2 = 0$$

$$\Rightarrow t(t - K) = -1$$

$$3x^2 + 4x + 2 = t = \frac{1}{K - 1}$$

For real roots, $D \geq 0 \Rightarrow \frac{3}{K-1} \geq 2 \Rightarrow \frac{(2K-5)}{(K-1)} \leq 0 \Rightarrow K \in \left(1, \frac{5}{2}\right]$

11. If two tangents drawn from a point P to the parabola $y^2 = 16(x-3)$ are at right angles, then the locus of point P is :

(1) $x + 4 = 0$ (2) $x + 2 = 0$ (3) $x + 3 = 0$ (4) $x + 1 = 0$

Ans. (4)

Sol. Locus is directrix of parabola

$$x - 3 + 4 = 0 \Rightarrow x + 1 = 0$$

12. The area of the region bounded by the parabola $(y - 2)^2 = (x - 1)$, the tangent to it at the point whose ordinate is 3 and the x-axis is :

(1) 4 (2) 6 (3) 9 (4) 10

Ans. (3)

Sol. Given $y = 3 \Rightarrow x = 2 \Rightarrow$ so point is (2, 3)

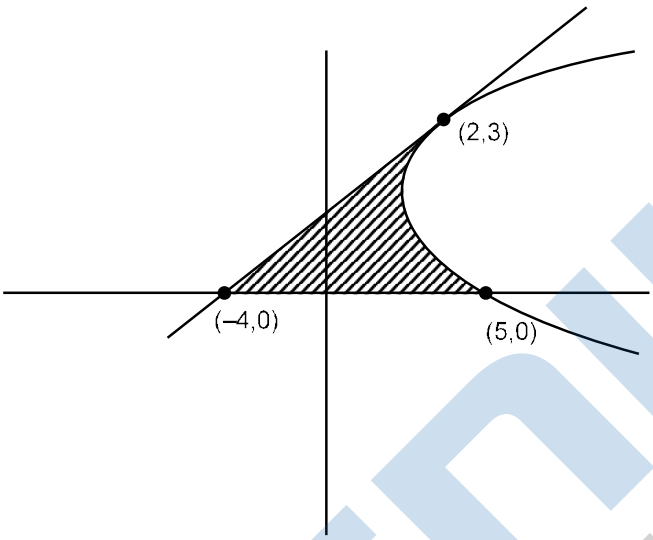
Differentiate given equation w.r.t.x, we get

$2(y-2)y' = 1$

$$y' = \frac{1}{2(y-2)}$$

$$y'|_{(2,3)} = \frac{1}{2}$$

so equation of tangent is $x - 2y + 4 = 0$



$$\text{Area} = \int_0^3 [(y-2)^2 + 1 - (2y-4)]dy$$

$$\text{Area} = \int_0^3 (y^2 - 6y + 9)dy$$

$$\text{Area} = \left(\frac{y^3}{3} - 3y^2 + 9y\right)_0^3 = 9 \text{ sq. unit}$$

13. Each of the persons A and B independently tosses three fair coins. The probability that both of them get the same number of heads is :

(1) $\frac{5}{8}$ (2) $\frac{1}{8}$ (3) $\frac{5}{16}$ (4) 1

Ans. (3)

Sol. {HHH, TTT, HHT, HTH, THH, HTT, THT, TTH}

$P = P_A(0H)\,P_B(0H) + P_B(1H)\,P_B(1H) + P_A(2H)\,P_B(2H) + P_A(3H)\,P_B(3H)$

$$P = \frac{1}{8}\times\frac{1}{8} + \frac{3}{8}\times\frac{3}{8} + \frac{3}{8}\times\frac{3}{8} + \frac{1}{8}\times\frac{1}{8}$$

$$P = \frac{5}{16}$$

14. Let M and m respectively be the maximum and minimum values of the function $f(x) = \tan^{-1}(\sin x + \cos x)$ in $\left[0, \frac{\pi}{2}\right]$. Then the value of tan(M – m) is equal to :

(1) $2-\sqrt{3}$ (2) $3-2\sqrt{2}$ (3) $3+2\sqrt{2}$ (4) $2+\sqrt{3}$

Ans. (2)

Sol. $f(x) = \tan^{-1}(\sin x + \cos x)$

$$1 \le \sin x + \cos x \le \sqrt{2}; x \in \left[0, \frac{\pi}{2}\right]$$

$$M = \tan^{-1}(\sqrt{2}), m = \tan^{-1}(1)$$

$$M - m = \tan^{-1}\left(\frac{\sqrt{2}-1}{1+\sqrt{2}}\right) = \tan^{-1}(3-2\sqrt{2})$$

$$\tan(m-m) = 3-2\sqrt{2}$$

15. The angle between the straight lines, whose direction cosines are given by the equations 2l + 2m – n = 0 and mn + nl + lm = 0, is :

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{2}$ (3) $\cos^{-1}\left(\frac{8}{9}\right)$ (4) $\pi - \cos^{-1}\left(\frac{4}{9}\right)$

Ans. (2)

Sol. $2\ell + 2m - n = 0$

$mn + n\ell + \ell m = 0$

$\ell m + n(\ell + m) = 0$

$\ell m + 2(\ell + m)^2 = 0$

$2\ell^2 + 2m^2 + 5\ell m = 0$

$$2\left(\frac{\ell}{m}\right)^2 + 2 + 5\left(\frac{\ell}{m}\right) = 0$$

Let $\frac{\ell}{m} = t$

$2t^2 + 5t + 2 = 0$

(i) $\frac{\ell}{m} = -2$

$\ell = -2m$

$n = -2m$

$(-2m, m, -2m)$

$(-2, 1, -2)$

(ii) $\frac{\ell}{m} = -\frac{1}{2} \Rightarrow m = -2\ell$

$n = -2\ell$

$(\ell, -2\ell, -2\ell)$

$(1, -2, -2)$

$$\cos\theta = \frac{-2-2+4}{9} = 0 \Rightarrow \theta = \frac{\pi}{2}$$

16. A differential equation representing the family of parabolas with axis parallel to y-axis and whose length of latus rectum is the distance of the point (2, –3) form the line 3x + 4y = 5, is given by :

(1) $11\frac{d^2x}{dy^2}=10$ (2) $11\frac{d^2y}{dx^2}=10$ (3) $10\frac{d^2y}{dx^2}=11$ (4) $10\frac{d^2x}{dy^2}=11$

Ans. (2)

Sol. $(x-\alpha)^2 = 4a\,(y-\beta)$

distance of point (2, –3) from line 3x + 4y = 5 is

$$4a=\left|\frac{6-12-5}{5}\right|$$

$$4a=\frac{11}{5}$$

$$\therefore (x-\alpha)^2=\frac{11}{5}(y-\beta)$$

Differentiate w.r.t.x

$$2(x-\alpha)=\frac{11}{5}\frac{dy}{dx}$$

$$2=\frac{11}{5}\frac{d^2y}{dx^2}$$

$$11\frac{d^2y}{dx^2}=10$$

17. Let [λ] be the greatest integer less than or equal to λ. The set of all values of λ for which the system of linear equations x + y + z = 4, 3x + 2y + 5z = 3, 9x + 4y + (28 + [λ])z = [λ] has a solution is :

(1) R (2) $(-\infty,-9)\cup[-8,\infty)$ (3) $(-\infty,-9)\cup(-9,\infty)$ (4) [–9, –8)

Ans. (1)

Sol. x + y + z = 4

3x + 2y + 5z = 3

9x + 4y + (28 + [λ])z = [λ]

$$d=\begin{vmatrix}1 & 1 & 1\\ 3 & 2 & 5\\ 9 & 4 & 28+[\lambda]\end{vmatrix}=-([\lambda]+9)$$

If [λ] + 9 ≠ 0 then unique solution

If [λ] + 9 = 0 then $D_1 = D_2 = D_3 = 0$ infinite solutions

So $\lambda \in R$

18. If $0 < x < 1$ and $y=\frac{1}{2}x^2+\frac{2}{3}x^3+\frac{3}{4}x^4+\ldots\ldots$, then the value of e^{1+y} at $x=\frac{1}{2}$ is :

(1) $\frac{1}{2}e^2$ (2) 2e (3) $2e^2$ (4) $\frac{1}{2}\sqrt{e}$

Ans. (1)

Sol. $y=\left(1-\frac{1}{2}\right)x^2+\left(1-\frac{1}{3}\right)x^3+\left(1-\frac{1}{4}\right)x^4+.....$

$=(x^2+x^3+x^4+.....)-\left(\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+.....\right)$

$=\frac{x^2}{1-x}+x-\left(x+\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+.....\right)=\frac{x}{1-x}+\ell n(1-x)$

Put $x=\frac{1}{2}\Rightarrow y=1-\ell n2$ Then $e^{1+y}=e^{1+1-\ell n2}=\frac{1}{2}e^2$

19. Let $A=\begin{pmatrix}[x+1] & [x+2] & [x+3]\\ [x] & [x+3] & [x+3]\\ [x] & [x+2] & [x+4]\end{pmatrix}$, where [t] denotes the greatest integer less than or equal to t. If del(A) = 192, then the set of values of x is in the interval :

(1) [62, 63) (2) [65, 66) (3) [60, 61) (4) [68, 69)

Ans. (1)

Sol. $A=\begin{bmatrix}[x+1] & [x+2] & [x+3]\\ [x] & [x+3] & [x+3]\\ [x] & [x+2] & [x+4]\end{bmatrix}$

$A=\begin{bmatrix}[x]+1 & [x]+2 & [x]+3\\ [x] & [x]+3 & [x]+3\\ [x] & [x]+2 & [x]+4\end{bmatrix}$

$R_1\to R_1-R_3,\ R_2\to R_2-R_3$

$A=\begin{bmatrix}1 & 0 & -1\\ 0 & 1 & -1\\ [x] & [x]+2 & [x]+4\end{bmatrix}$

det (A) = 1 ([x] + 4 + [x] + 2) –1 (–[x])

= 3[x] + 6

det (A) = 192 = 3[x] + 6

3[x] = 186

[x] = 62

$x\in[62, 63)$

20. Let A(a, 0) B(b, 2b + 1) and C(0, b), $b\neq 0$, $|b|\neq 1$ be points such that the area of triangle ABC is 1 sq. unit, then the sum of all possible values of a is :

(1) $\frac{-2b}{b+1}$ (2) $\frac{2b^2}{b+1}$ (3) $\frac{-2b^2}{b+1}$ (4) $\frac{2b}{b+1}$

Ans. (3)

Sol. A(a, 0), B(b, 2b + 1), C(0, b)

$$\text{Area} = \left|\frac{1}{2}\begin{vmatrix} a & 0 & 1 \\ b & 2b+1 & 1 \\ 0 & b & 1 \end{vmatrix}\right| \Rightarrow \frac{1}{2}[a(b+1)+b^2] = \pm 1$$

$$a = \frac{2-b^2}{b+1} = \frac{-2-b^2}{b+1}$$

Sum of possible values of $a = \frac{-2b^2}{b+1}$

Numeric Value Type

This Section contains **10 Numeric Value Type question**, out of 10 only 5 have to be done.

1. Let S be the sum of all solutions (in radians) of the equation $\sin^4\theta + \cos^4\theta - \sin\theta - \cos\theta = 0$ in: $[0, 4\pi]$. then $\frac{8S}{\pi}$ is equal to _____.

Ans. (56)

Sol. $\sin^4\theta + \cos^4\theta - \sin\theta\cos\theta = 0 \qquad \theta \in [0, 4\pi]$

$\Rightarrow \quad 1 - 2\sin^2\theta\cos^2\theta - \sin\theta\cos\theta = 0 \Rightarrow 2 - \sin^2 2\theta - \sin 2\theta = 0$

$\Rightarrow \quad \sin^2 2\theta + \sin 2\theta - 2 = 0 \Rightarrow \sin^2 = 1\ 2\ \theta \in [0, 8\pi]$

$$\theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}$$

$\Rightarrow$ Sum of the solutions = $S = \frac{28\pi}{4}$

Then value of $\frac{8S}{\pi} = 8 \times \left(\frac{28\pi}{4\pi}\right) = 56$

2. Let A $(\sec\theta, 2\tan\theta)$ and B$(\sec\phi, 2\tan\phi)$, where $\theta + \phi = \pi/2$, be two points on the hyperbola $2x^2 - y^2 = 2$. If (α, β) is the point of the intersection of the normals to the hyperbola at A and B, then $(2\beta)^2$ is equal to _____.

Ans. (Bonus)

Sol. Let A $(\sec\theta, 2\tan\theta)$ lie on hyperbola

$2x^2 - y^2 = 2$

$2(1 + \tan^2\theta) - 4\tan^2\theta = 2$

$2\tan^2\theta = 0$

$\tan\theta = 0$

Similarly point B$(\sec\theta, 2\tan\phi)$

$\tan\phi = 0$

but in Question given that $\theta + \phi = \frac{\pi}{2}$

which is not possible

3. Let S = {1,2,3,4,5,6,9}. Then the number of elements in the set T = {A $\subseteq$ S : A $\neq \phi$ the sum of all the elements of A is not a multiple of 3} is _____.

Ans. (80)

Sol. 3, 6, 9 (3n type)

2, 5 ((3n – 1) type)

1, 4 ((3n – 2) type)

Let N_p = Number of subsets of S containing p elements which are not divisible by = 3

For p = 1

$^2C_1 + {}^2C_1 = 4$

For p = 2

$^3C_1\ {}^2C_1 + {}^3C_1\ {}^2C_1 + {}^2C_2 + {}^2C_2 = 14$

For p = 3

$^3C_1(^2C_2 + {}^2C_2) + {}^3C_2(^2C_1 + {}^2C_1) + {}^2C_2\ {}^2C_1 + {}^2C_1\ {}^2C_2 = 22$

For p = 4

$^3C_1[(^2C_2\ {}^2C_1) + (^2C_1\ {}^2C_2)] + {}^3C_2(^2C_2 + {}^2C_2) + {}^3C_3\ (^2C_1 + {}^2C_1) = 22$

For p = 5

$^3C_2[(^2C_2\ {}^2C_1) + (^2C_1\ {}^2C_2)] + {}^3C_3\ (^2C_2 + {}^2C_2) = 14$

For p = 6

$^3C_3[^2C_2\ {}^2C_1) + (^2C_1\ {}^2C_2)] = 4$

$\Rightarrow$ Total subsets satisfying given condition = 4 + 14 + 22 + 22 + 14 + 4 = 80

4. Two circles each of radius 5 units touch each other at the point (1, 2). If the equation of their common tangent is 4x + 3y = 10, and $C_1(\alpha, \beta)$ and $C_2(\gamma, \delta)$, $C_1 \neq C_2$ are their centres, then $|(\alpha + \beta)(\gamma + \delta)|$ is equal to _____.

Ans. (40)

Sol. $(x-1)^2 + (y-2)^2 + \lambda(4x + 3y - 10) = 0$

$$x^2 + y^2 + (2\lambda - 1)2x + \left(\frac{3}{2}\lambda - 2\right)2y + 5 - 10\lambda = 0$$

$$r = \sqrt{(2\lambda-1)^2 + \left(\frac{3}{2}\lambda - 2\right)^2 - (5 - 10\lambda)} = 5$$

$$4\lambda^2 + 1 - 4\lambda + \frac{9}{4}\lambda^2 + 4 - 6\lambda - 5 + 10\lambda = 25$$

$$\frac{25}{4}\lambda^2 - 25 = 0$$

$\lambda = \pm 2$

for $\lambda = 2$

$x^2 + y^2 + 6x + 2y - 15 = 0$; $C_1(-3, -1)$

for $\lambda = -2$

$x^2 + y^2 - 10x - 10y + 25 = 0$; $C_2(5, 5)$

$|(\alpha + \beta)(\gamma + \delta)| = 40$

5. $3 \times 7^{22} + 2 \times 10^{22} - 44$ when divided by 18 leaves the remainder_____.

Ans. (15)

Sol. $3 \times (6+1)^{22} + 2 \times (9+1)^{22} - 44 = 18\lambda + 3 + 18\mu + 2 - 44$

$= 18\delta - 39$

$= 18\beta + 15 \Rightarrow$ remainder = 15

6. An online exam is attempted by 50 candidates out of which 20 are boys. The average marks obtained by boys is 12 with a variance 2. The variance of marks obtained by 30 girls is also 2. The average marks of all 50 candidates is 15. If μ is the average marks of girls and σ^2 is the variance of marks of 50 candidates, then $\mu + \sigma^2$ is equal to _____.

Ans. (25)

Sol. 20 Boys, 30 Girls

$\sigma_B^2 = 2, \bar{X}_B = 12, \sigma_G^2 = 2$

$$\bar{X}_G = \frac{50 \times 15 - 20 \times 12}{30} = \frac{510}{30} = 17 = \mu$$

Variance of 50 candidates

$$\sigma^2 = \frac{20\sigma_B^2 + 30\sigma_G^2}{20+30} + \frac{20 \cdot 30}{(20+30)^2}\left(\bar{X}_B - \bar{X}_G\right)^2$$

$$= \frac{20 \times 2 + 30 \times 2}{50} + \frac{600}{2500}(25) = 8$$

$\mu + \sigma^2 = 17 + 8 = 25$

7. $\int \frac{2e^x + 3e^{-x}}{4e^x + 7e^{-x}} dx = \frac{1}{14}(ux + v\log_e(4e^x + 7e^{-x})) + C$, where C is a constant of integration, then u + v is equal to _____.

Ans. (7)

Sol. $$I = \int \frac{2e^x + 3e^{-x}}{4e^x + 7e^{-x}} dx = \int \frac{2e^x + 3}{4e^{2x} + 7} dx$$

Here $2e^{2x} + 3 = A(8e^{2x}) + B(4e^{2x} + 7)$

$\Rightarrow$ $2e^{2x} + 3 = (8A + 4B)e^{2x} + 7B$

$\Rightarrow$ $B = \frac{3}{7}$ & $A = \frac{1}{28}$

$$I = \int \frac{\frac{1}{28}(8e^{2x}) + \frac{3}{7}(4e^{2x} + 7)}{4e^{2x} + 7} dx$$

$$I = \frac{1}{28}\ln|4e^{2x} + 7| + \frac{3}{7}x + C$$

$$I = \frac{1}{28}\ln|e^x(4e^x + 7e^{-x}|) + \frac{3}{7}x + C$$

$$\Rightarrow u = \frac{13}{2} \text{ \& } v = \frac{1}{2}$$

$$\Rightarrow u + v = \frac{13}{2} + \frac{1}{2} = 7$$

8. Let z_1 and z_2 be two complex numbers such that $\arg(z_1 - z_2) = \frac{\pi}{4}$ and z_1, z_2 satisfy the equation $|z - 3| = \text{Re}(z)$. Then the imaginary part $z_1 + z_2$ is equal to _____.

Ans. (6)

Sol. Let $z_1 = x_1 + iy_1$, $z_2 = x_2 + iy_2$

$z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2)$

$\because \arg(z_1 - z_2) = \frac{\pi}{4} \Rightarrow \tan^{-1}\left(\frac{y_1 - y_2}{x_1 - x_2}\right) = \frac{\pi}{4}$

$\Rightarrow y_1 - y_2 = x_1 - x_2$ ------(1)

$|z_1 - 3| = \text{Re}(z_1) \Rightarrow (x_1 - 3)^2 + y_1^2 = x_1^2$

$|z_2 - 3| = \text{Re}(z_2) \Rightarrow (x_2 - 3)^2 + y_2^2 = x_2^2$

$\Rightarrow (x_1 - 3)^2 - (x_2 - 3)^2 + (y_1^2 - y_2^2) = x_1^2 - x_2^2$

$\Rightarrow (x_1 - x_2)(x_1 + x_2 - 6) + (y_1 - y_2)(y_1 + y_2) = (x_1 + x_2)(x_1 - x_2)$

$\Rightarrow x_1 + x_2 - 6 + y_1 + y_2 = x_1 + x_2$

$\Rightarrow y_1 + y_2 = 6$

9. Let S be the mirror image of the point Q(1,3,4) with respect to the plane $2x - y + z + 3 = 0$ and let $R(3, 5, \gamma)$ be a point of this plane. Then the square of the length of the line segment SR is _____.

Ans. (72)

Sol. Let point $S(\alpha, \beta, \gamma)$

$\frac{\alpha - 1}{2} = \frac{\beta - 3}{-1} = \frac{\gamma - 4}{1} = \frac{-2(2 - 3 + 4 + 3)}{4 + 1 + 1} = -2$

$\Rightarrow S(\alpha, \beta, \gamma) = (-3, 5, 2)$

and point $R(3, 5, \gamma)$ lies on the plane $2x - y + z + 3 = 0$

$\Rightarrow 6 - 5 + \gamma + 3 = 0 \Rightarrow \gamma = -4$

$\Rightarrow R(3, 5, -4)$ & $S(-3, 5, 2)$

$SR = \sqrt{36 + 0 + 36} \Rightarrow (SR)^2 = 72$

10. The probability distribution of random variable X is given by :

x	1	2	3	4	5
P(X)	K	2K	2K	3K	K

Let $p = P(1 < X < 4 | X < 3)$. If $5p = \lambda K$, then λ is equal to _____.

Ans. (30)

Sol. $\Sigma P(x) = 1$

$\Rightarrow k + 2k + 2k + 3k + k = 1$

$$k = \frac{1}{9}$$

$$p = P(1 < x < 4 \mid x < 3) = \frac{p(x = 2)}{p(x < 3)}$$

$$p = \frac{\frac{2k}{9k}}{\frac{k}{9k} + \frac{2k}{9k}} = \frac{2}{3}$$

$5p = \lambda k$

$$5 \times \frac{2}{3} = \lambda \times \frac{1}{9} \Rightarrow \lambda = 30$$